**Assignment 2**

**Name: Mingyuan Cui**

**StudentID: u5323288**

**Question 1**

P is short for the property.

**Induction Start**

If A|=B and A is suitable for T.

∵There is no ¬ in the formula

∴B[B/T]=T

T is a tautology

∴A|=T, A|=B[B/T].

∴P holds for atom B.

**Induction Step**

a. Assume that P holds for arbitrary formulas F and G: A|=F, A|=G, A is suitable for T, and A|=F[B/T], A|=G[B/T].

b. A|=F∧G →A|=F and A|=G,

A|=F[B/T] and A|=G[B/T],

∴A|=(F[B/T]∧G[B/T]),

∵F[B/T]∧G[B/T]↔(F∧G)[B/T]

∴A|=(F∧G)(B/T)

∴P holds for F∧G

c. A|=F∨G →A|=F∨A|=G,

A|=F[B/T] or A|=G[B/T],

∴A|=(F[B/T]∨G[B/T]),

∵F[B/T]∨G[B/T]↔(F∨G)[B/T]

∴A|=(F∨G)[(B/T)

∴P holds for F∨G

∴ If A |= F and A is suitable for T then A |= F[B=T ], where F[B=T ] is the formula obtained from F by replacing every occurrence of B by T.

**Question 2**

Theorem 25 (Compactness) claims that a clause set M` is unsatisfiable iff some finite subset of M is unsatisfiable. Thus, we can infer that a clause set M` is satiffaiale iff every finite subset of M is satisfiable.

The resolvent of the first two clauses is B1∨ B2

The resolvent of the fisrt three clauses is A2∨ B2

…

The resolvent of the first n clauses is:

a.An-1∨Bn-1,whenn is even.

b.Bn-2∨An-1,when n is odd.

The resolvent can be easily satisfied by just making either side true.

Therefore we can always find a model of such finite subsets in this form by just making its resolvent satisfiable.

Every finite subset of M must be a finite subset of the first n clauses I mentioned above.

And every finite subset of M is satisfiable according to the inference of Theorem 25.

Hence M is satisfiable.

**Question 3**

∀x∃y((R(y,x)∧¬R(x,y))∨∃yQ(x,y))

⇔∀x∃y((R(y,x)∧¬R(x,y))∨∃zQ(x,z))

⇔∀x∃y∃z((R(y,x)∧¬R(x,y))∨Q(x,z)) (Prenex Normal Form)

⇔∀x ((R(f(x),x)∧¬R(x,f(x)))∨Q(x,h(x))) (Skolemized Form)

⇔∀x ((R(f(x),x)∨Q(x,h(x)))∧(¬R(x,f(x)) ∨Q(x,h(x))))

⇔{(R(f(x),x)∨Q(x,h(x))),(¬R(y,f(y)) ∨Q(y,h(y)))} (CNF)

**Question 4**

1.

Herbrand universe is

D(F)={a,b,f(a),f(b),f(f(a)),f(f(b))…}

Herbrand expansion is as below.

P(a)∧¬P(f(b))∧(¬P(a)∨P(f(a))) mit [x/a]

P(a)∧¬P(f(b))∧(¬P(b)∨P(f(b))) mit [x/b]

P(a)∧¬P(f(b))∧(¬P(f(a))∨P(f(f(a)))) mit [x/f(a)]

P(a)∧¬P(f(b))∧(¬P(f(f(a)))∨P(f(f(f(a))))) mit [x/f(f(a)]]

…

E(F)={P(a), ¬P(f(b))}{¬P(a)∨P(f(a)), ¬P(b)∨P(f(b)), ¬P(f(a))∨P(f(f(a))), ¬P(f(f(a)))∨P(f(f(f(a))))…}

2.

Assume that A is a herbrand structure for F, and A|=E(F)

If E(F) is satisfiable, then every finite subset of E(F) should be satisfiable according to Theorem 25(compactness)

Then we can infer from above

P(a) T P(f(b)) F P(f(a)) T P(b) F P(f(f(a)) T P(f(f(b))) F…

We can find that every atom with occurrence of a is true and every atom with occurrence of b is false. Therefore a≠b .

Herbrand Base of F is

{P(a),P(b),P(f(a)),P(f(b)),P(f(f(a))),P(f(f(b)))…}

A suitable herbrand interpretation of F is

{ P(a),¬P(b),P(f(a)),¬P(f(b)),P(f(f(a))),¬P(f(f(b)))…}

A herbrand structure A that A|=F is

{ a, f(a), f(f(a))…}

∵A|=E(F)

∴A|=F (Lemma 48)

**Question 5**

1.

U={x=y, g(f(x))=g(y)}

U={g(f(y))=g(y)}Binding σ={x→y}

U={f(y)=y}Decomposition

U={ y=f(y) } Orientation

y occurs in f(y),thus FAIL.

2.

U={a=x, g(f(x, z))=y, y=g(f(z, x))}

U={x=a, g(f(x, z))=y, y=g(f(z, x))} Orientation

U={x=a, y=g(f(x, z)), y=g(f(z, x))} Orientation

U={y=g(f(a, z)), y=g(f(z, a))} Binding σ={x→a}

U={g(f(a, z))=g(f(z, a))} Binding σ={ x→a , y→g(f(a, z))}

U={f(a, z)=f(z, a)} Decomposition

U={a=z ,z=a} Decomposition

U={z=a ,z=a} Orientation

U={a=a } Binding σ={ x→a , y→g(f(a, z)), z→a}

U={ }

σ={ x→a , y→g(f(a, z)), z→a}

**Question 6**

(1)

{y/f(x)}

(1) (3)

{x/a}

(2)

{x/f(a)}

(4)

{}

As the final result is NIL, a resolution refutation of the clause set is found.